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CAUSALITY AND SPACETIME STRUCTURE IN RELATIVITY

1

The modern idea of causality, dominant since the seventeenth century, contrasts sharply with the earlier acceptation of the term *cause* (*causa, aitía*) in the Aristotelian tradition. While the Aristotelian physicist sought for the causes of *things*, and admitted as such anything, be it stuffs or structures, drives or goals, that might reasonably account for the object under study, the modern scientist inquires after the causes of *processes* or *states of affairs*, meaning the agents that bring them about. This drastic change in the sense and scope of one of the fundamental categories of scientific thought has often been said to stem from a change of purpose: modern science, it is said, pursues the domination, not the contemplation of nature. With such an end in mind, the scientist would tend of course to ignore nature's own ends – if indeed it has any – and to view each natural stuff as nothing but an aggregate of passive and active dispositions to aid or resist man's aims. However, though the prospective "master and lord of nature"[1] might thus free himself from the Aristotelian preoccupation with the so-called final and material causes, it is unlikely that he could direct all his attention to "sources of change" (*arkhai tes metaboles*) or efficient causes, in utter disregard of form or structure. For efficient causes can only be grasped as such in a context that must be structurally conceived. It is true that the experimental method characteristic of modern scientific inquiry is primarily fit for disclosing the presence and mode of operation of natural agents. But the results of experiment can only make sense if they and the experimental setting that yields them are described and understood in structural terms. Why then has 'cause', in modern parlance, become synonymous with 'efficient cause', while the idea of 'formal causation' and explanation by structure, though alive and strong in actual scientific practice, has all but vanished from 'metascientific' discouse? A brief reflection on the research program sketched in the Preface to Newton's *Principia* can throw some light on this question. Newton expected natural philosophy to find out the forces of nature by studying the phenomena of motion, and then, from those forces, to infer the other phenomena.[2] The program's ultimate goal is therefore

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to derive the course of nature from the acting sources of change. Natural philosophy is thus clearly understood as a quest not just for causes in general, in the Aristotelian and medieval sense, but specifically for *efficient* causes, powers, forces. The latter, however, must be gathered from a particular type of changes, namely, motions, i.e. changes of place in time. But motions can only be described in terms of the twofold mathematical structure of Newtonian time and space (which are assumed to be isomorphic, respectively, to the real number field $\mathbf{R}$, and to $\mathbf{R}^3$, endowed with the standard metric) – or, as we would rather put it nowadays, in terms of the single mathematical structure of Newtonian spacetime (homeomorphic with $\mathbf{R}^4 = \mathbf{R} \times \mathbf{R}^3$). Hence, the new natural philosophy does not neglect what Newton thought was the structural background of physical events, but rather takes it for granted as a prerequisite for grasping the causal relations which Newton's program aims at discovering. The close connection between causality and spacetime is a fundamental premise of the physics of central forces that prevailed in the heyday of Newtonianism in the eighteenth and early nineteenth centuries. According to this view – illustrated, for instance, by Coulomb's electrostatics and Ampère's electrodyanmics – the forces of nature depend on the changing positions of their sources in space, so that causality is subordinate to geometry.

In an important manuscript 'On the Gravity and Equilibrium of Fluids', published by the Halls in 1962, Newton expressly characterized space and time as pure structures or relational systems: "It is only through their mutual order and positions that the parts of time and space can be seen to be the very same which they are in fact. They do not have any other individuation principle besides the said order and positions, which therefore cannot change."[3] Such structures have their own peculiar mode of existing (*quendam sibi proprium existendi modum*) and do not fall under the categories of substance and accident into which traditional ontology classified all beings. They are not substances, for they do not underlie any actions, such as a mind's thoughts or a body's motions. Nor are they accidents, for we can clearly conceive them as existing without any substrate, "as when we imagine spaces outside the world or places empty of body".[4]

Philosophers have seldom felt at ease with Newtonian space and time. Being for the most part less daring and open-minded than Newton, they have been unwilling to accept that something can exist which is neither a substance, nor an attribute or relation of substances, and which therefore cannot be described in the established Aristotelian terminology. Even such a bold thinker as Kant could only reconcile himself with Newtonian space

and time by regarding them as "mere forms" of human sensibility and degrading everything that is contaminated with them to the status of second-rank, phenomenal being. We need not recall here the numerous attempts to develop a 'relational' theory of physical space and time, whereby these structures or relational systems would no longer be viewed as self-sustained – or as sustained by God alone – as in Newton, but as an abstract reflection of the actual network of changing relations between material things. Such attempts were revitalized in this century by Einstein's profound revision of our ideas of space and time. It was generally thought that the relativistic conception of spacetime lent strong support to the relationist school, at least until Adolf Grünbaum (1957) issued a warning about "the philosophical retention of absolute space" in General Relativity. More precisely, it was believed that spacetime structure, as countenanced by Einstein's theory, was no more than an outline – so to speak, an intellectual shadow – of the system of causal relations between real events. According to Hans Reichenbach, this "causal theory of space and time" is "the philosophical result of the theory of relativity".[5] "The combined space-time order reveals itself as the ordering schema of causal chains (*Ordnungsschema der Kausalreihen*), and thus as the expression of the causal structure of the universe ... The order of causal chains is ultimately reflected in all space-time determinations."[6]

I intend here to analyze and evaluate these claims in connection with Special and General Relativity. As we shall see, they cannot mean quite the same in both contexts. But before proceeding with this task, let us take a look at Reichenbach's statements in their abstract generality.[7] To a traditional empiricist, who believes that all scientific knowledge is to be attained by induction from experimental results, they must sound inherently plausible. He might argue as follows: Relativity does not take space and time for granted; the Special Theory criticizes the Newtonian view of them; the General Theory binds the spacetime structure of the world to the dynamics of gravity; thus spacetime geometry becomes a matter of experimental inquiry; but experiments are properly directed at disclosing causal relations; hence, the structure of spacetime – insofar as it is not a factitious and freely disposable convention – ought to reflect the causal system of nature ('causal' being of course understood in its restricted modern sense). In the light of the foregoing argument we can readily see why the causal theory of spacetime is favored by inductivist empiricists. We can also surmise why its advocates are wont to season it with a dash or two of chronogeometric conventionalism – namely, in order to cover those features of spacetime geometry which apparently cannot be accounted for by the causal theory. But the argument

also implies that, if the causal view of relativistic spacetime is untenable or can only be held in a Pickwickian sense, Relativity is incompatible with inductivism.

2

We turn now to the causal interpretation of flat relativistic spacetime, i.e. the spacetime of Special Relativity. To understand its claims and motivation we must recall the main features of Minkowski's geometrical formulation of the theory. The standard practice of labelling physical events with real number quadruples that vary smoothly as the argument ranges over space and time presupposes that events themselves or their 'locus' or 'arena' constitute a four-dimensional differentiable manifold. Minkowski called this manifold "*die Welt*" – "the world" – but we prefer to call it "spacetime". The (special) Principle of Relativity, stated by Einstein in 1905, implies that the spacetime manifold admits an atlas of global charts with the following property: the laws of physics take the same mathematical form when referred to any of these charts. The charts, which we shall call *Lorentz charts*, combine a set of Cartesian space coordinates bound to an inertial rigid frame, with the Einstein time coordinate defined by means of bouncing light signals, as explained in Section 1 of 'Zur Elektrodynamik bewegter Körper'.[8] The postulated form invariance of physical laws under coordinate transformations from one Lorentz chart to another obtains of course only if the same standard units of time and length are employed in the construction of all Lorentz charts. The general form of such coordinate transformations can be readily ascertained if we assume with Einstein that they preserve in particular the two following laws: the Law of Inertia and the Principle of the Constancy of the Speed of Light (regardless of the state of motion of its source). If x and y are two Lorentz charts with the same origin (i.e. if x and y send the same spacetime point to $0 \in \mathbf{R}^4$), the coordinate transformation $x \cdot y^{-1}$ is demonstrably a Lorentz transformation, i.e. a linear permutation of $\mathbf{R}^4$ which can be characterized as follows: choose, for simplicity's sake, the standard units employed in the definition of Lorentz charts so that the constant speed of light in vacuo equals 1; then, the matrix A of any Lorentz transformation satisfies the equation $A^{\mathrm{T}} HA = H$, where A^{T} is the transpose of A and H is the diagonal matrix $(-1, 1, 1, 1)$. If x and y do not share the same origin, $x \cdot y^{-1}$ is a so-called Poincaré transformation, i.e. the product of a Lorentz transformation and a translation. Lorentz transformations obviously form a group.[9] Consequently, the Poincaré

transformations, being generated by the Lorentz group and the group of translations, form a group too. The Poincaré group acts transitively and effectively on $\mathbf{R}^4$ through the coordinate transformations between Lorentz charts. Since the latter are global, each coordinate transformation $x \cdot y^{-1}$ between two arbitrary Lorentz charts x and y is matched by a unique point transformation $x^{-1} \cdot y$ of spacetime onto itself. Through these point transformations the Poincaré group acts transitively and effectively on the spacetime manifold (as conceived by Special Relativity). This is the key to Minkowski's geometrical reading of Einstein's theory. According to Felix Klein's Erlangen Program, the action of a group on a manifold endows the latter with a geometric structure characterized by the group invariants. The Minkowski geometry induced on the spacetime manifold by the stated action of the Poincaré group is contained, so to speak, in a nutshell in a single two-point invariant that I shall call the *separation* between spacetime points. Let P and Q be two such points and let (P^i) and (Q^i) denote their respective coordinates in any given Lorentz chart; their separation is then given by

$$|P - Q| = \Sigma_i \Sigma_j \eta_{ij} (P^i - Q^i)\ (P^j - Q^j),$$

where i and j range over $\{0, 1, 2, 3\}$ and the η_{ij} are the elements of the matrix H (i.e. $\eta_{00} = -1$; $\eta_{ii} = 1$ if $i \neq 0$; $\eta_{ij} = 0$ if $i \neq j$).[10] We say that P and Q are *separate* if $|P - Q| > 0$, and *connected* if $|P - Q| \leqslant 0$. It will be noted that both relations are symmetric (since $|P - Q| = |Q - P|$). All points separate from a given point P lie outside a two-sheet (hyper-)cone with its vertex at P. For any point Q on this cone $|P - Q| = 0$, which is why it is called the *null-cone* at P (*Null* being the German word for *zero*). Poincaré point-transformations map null-cones onto null-cones. We can choose one of the two sheets of the null-cone at a point P and call it the 'future' null-cone at P. Our choice is then propagated unambiguously by translation to all other null-cones. A point transformation that preserves the choice of a 'future' is said to be orthochronous. Orthochronous Poincaré (resp. Lorentz) transformations form a subgroup of the Poincaré (resp. Lorentz) group.

The causal interpretation of Minkowski geometry stems from two simple facts concerning separateness and connectedness between spacetime points:

(i) The relation 'greater than' between the time coordinates of two spacetime points P and Q is preserved by all orthochronous Poincaré transformations if P and Q are connected, but it is not generally preserved by such transformations if P and Q are separate. If, as was commonly assumed before the advent of General Relativity, admissible time coordinates reflect the

'real', 'objective' time order between events, it is clear that such a time order can subsist only between events at connected points, but not between events at separate points.

(ii) The physical conditions governing Lorentz charts imply that two separate spacetime points P and Q can only be joined by a signal travelling faster than light (relatively to inertial reference frames endowed with Einstein time).

Combining these two facts, we see that any faster-than-light signal, i.e. any transfer of energy and momentum with speed $|dr/dt| > 1$ (where t and r respectively denote the time and the length of the position vector in an arbitrary Lorentz chart), must be undetermined as to its origin and destination, for its emission relative to a Lorentz chart turns out to be its reception relative to another one, and vice versa. If such indeterminateness is repugnant to our metaphysical insights or feelings we are bound to conclude that all transfers of energy and momentum occur at a speed equal to or less than that of light, so that physical influence can only be exerted between connected spacetime points. It follows that *events at separate points are always causally unrelated*, while *causally connected events must take place at connected points*.

The preceding considerations bestow a clear causal meaning on the Minkowski geometry but do not by themselves provide it with a purely causal foundation. There is apparently no reason to expect that the spacetime relations of separateness and connectedness, which have thus been given a causal meaning, will encompass or generate the entire system of chrono-geometric predicates. However, this important and seemingly unlikely result can be extracted, after some conceptual reshuffling, from the axiomatic formulation of Minkowski geometry published by A. A. Robb in 1914. Robb's work was overlooked by Reichenbach in the 1920's, but was put to good use in Henryk Mehlberg's 'Essai sur la théorie causale du temps' (1935/37). Mehlberg was able to derive the spacetime geometry of Special Relativity from a set of axioms with a single primitive term, namely, a binary predicate whose intended meaning is precisely that of our "connectedness", but which Mehlberg unabashedly called "*le rapport causal*". We ought not to set much store by such feats of syntactic economy. Usually, the complexity of unproven propositions does more than compensate for the paucity of undefined terms. For instance, Mario Pieri (1899) built Euclidean geometry from the sole concept of 'motion', but the motions he speaks about are so restricted by his twenty axioms that they turn out to be precisely the Euclidean motions. Similarly, Mehlberg's 'causal relation' is not the general,

fairly neutral notion that physicists and philosophers associate with this expression, for Mehlberg's axiom system has loaded it with the full connotation of connectedness in Minkowski spacetime. There is, however, a deep and powerful reason why Mehlberg's – and Robb's – achievement is more substantial than Pieri's. If we compare Mehlberg's "*rapport causal*", i.e. connectedness, with the fundamental invariant of Minkowski geometry, namely, the numerical separation between spacetime points, it will look as if the latter's invariance were a much stiffer requirement than the former's. After all, connectedness holds between two spacetime points when their separation takes *any* non-positive value, and there are uncountably many such values. And yet the group of point transformations that preserve connectedness – the so-called "causal group" – is only slightly (one is tempted to say, insignificantly) larger than the Poincaré group – which is a subgroup of it. As E. C. Zeeman (1964) has shown, the causal group of Minkowski spacetime is generated by the Poincaré group and the group of dilatations (i.e. the spacetime permutations that multiply all separations by a positive constant factor). There are of course uncountably many dilatations too – they add a whole dimension to the Poincaré group. But one would tend to agree that this addition is trivial and lacks a genuine geometrical import. Thus, Zeeman's Theorem, together with the aforesaid causal interpretation of chronogeometrical connectedness, apparently suffice to establish that, in a universe governed by Special Relativity, the spacetime structure does no more than reflect the system of causal relations. And yet, as I shall now show, several arguments can be put forward which undermine the strength or curtail the scope of this contention.

In the first place, let me note that, while the causal group of Minkowski spacetime does not differ *geometrically* from the Poincaré group in a significant way, this does not mean, however, that their difference is negligible also from a *physical* point of view. Let F and $T(F)$ be two configurations in Minkowski spacetime, such that $T(F)$ is the image of F by the Poincaré point transformation T. Then, according to the (special) Principle of Relativity, two deterministic experiments whose initial conditions are represented by F and $T(F)$, respectively, must have similar outcomes – more precisely, the outcome of the latter must be the transform by T of the outcome of the former. But if T is a typical element of the causal group, and hence the product of a Poincaré transformation and a dilatation, there need not be such a neat correspondence between both experiments. (A *coordinate* dilatation amounts indeed to a conventional change of units; but a *point* dilatation involves a real rescaling of physical things and processes, which, given the

variety of ways in which times and lengths occur in the laws of nature, obviously cannot be trivial.)

In the second place, we ought to bear in mind that in the proposed causal interpretation separateness and connectedness are *modalized* causal predicates: two causally related events *necessarily* take place at connected points; their spacetime locations *cannot* be separate.[11] This does not mean, however, that every connected pair of points is actually joined by a causal chain; indeed, for all we know, they might not even be the site of any events at all. (Minkowski avoided the latter alternative by conceiving spacetime as a densely packed plenum of events;[12] under the field laws of classical physics each event would then influence or be influenced by whatever happened at locations connected with its own.) Thus, chronogeometrical connectedness is not an abstract distillation of concrete causal connections, but rather a prerequisite which the latter must fulfil. Spacetime structure, more than a reflection of actual causal relations, turns out to be a norm by which such relations are constrained.

In the third place, it must be emphasized that the interpretation of spacetime connectedness and separateness as modalized causal concepts does not logically follow from Special Relativity and is not supported by its empirical evidence, but is motivated by philosophical sentiments. Special Relativity does indeed entail that any object moving with the speed of light in an inertial frame travels with the same speed relative to every inertial frame; and that no material object can be accelerated from rest in such a frame to a velocity equal to or greater than that of light. This implies, in turn, that only connected point pairs can be joined by the worldlines (i.e. the spacetime trajectories) of ordinary matter or radiation. But Special Relativity does not preclude the existence of physical objects that always move faster than light in every inertial frame. Hypothetical particles with this property were extensively discussed in the 1960's under the name of 'tachyons', but have not been detected hitherto. According to Special Relativity, a tachyon would be in many ways peculiar, but not inherently absurd. (For instance, if a particle moves with velocity $v > 1$, the Lorentz factor $\gamma = 1/\sqrt{1 - v^2}$ is an imaginary number. Consequently, such a particle can only be assigned a measurable, and hence real-valued, energy $\mu\gamma$ and momentum $\mu v\gamma$ if its 'rest mass' μ is imaginary-valued. This is strange, but consistent with the fact that nobody will every find the particle at rest in his lab.) Since all events in a tachyon's history occur at mutually separate spacetime points they are subject to the sort of temporal ambiguity that I mentioned earlier: the tachyon's emission in an inertial frame may be its absorption relative to

another one, and vice versa. Such ambiguity would mean indeed that Einstein time is more deeply at variance with Newtonian time than was initially believed, but is otherwise credible. After sixty or more years of General Relativity we know enough about the physical meaning of coordinates to accept even wilder changes of description as a consequence of coordinate transformations. One may still feel inclined to ask which is the 'real' time order of tachyonic events; but can we be so sure that this category is universally applicable – that it is not solely appropriate to the like of us, slow and heavy creatures, with only a modest power of chronometric resolution? Genuine paradoxes would no doubt ensue if we could use tachyons to change our past. Let A and B be two connected spacetime points, e.g. in my own life history, and let C be another point, separate from A and B. If energy and momentum can be transferred through tachyons there could be a closed causal chain from A to B to C to A. Might I not then attempt to change events at A from B even though B lies to A's future? In my view, such an attempt would not only run against common sense, but also against the very notion of a closed causal chain. If someone is caught in a closed causal chain he is thereby deprived of every chance of altering it. This may be beneath our presumed human dignity but it is certainly compatible with commonly accepted ideas about physical existence.

The fact that tachyons are compatible with Special Relativity, even if they do not happen to exist, implies that the distinction between connected and separate spacetime points does not by itself provide a classification of event pairs into those that are causally connectible and those that are not; but corresponds rather to a distinction between two conceivable forms of causal connectibility, namely, through tachyons and through massive matter and radiation, of which the former, as far as we can tell, is not realized in nature. This conclusion agrees well with the empirical foundations on which Special Relativity was originally based. They did not include a statistical sampling of event pairs, showing which were and which were not causally connected; but consisted of evidence for the validity of the Maxwell–Lorentz equations of electrodynamics in every inertial frame ('coordinatized' by a Lorentz chart), and in particular for the sameness of the speed of light *in vacuo* in all such frames. The invariant speed of light was then demonstrably the least upper bound of the speed of ordinary, massive matter relative to inertial frames, which implied, in turn, the said distinction between two modes of propagation of causal influence.

Finally, I should point out that the causal theory of spacetime cannot account for what is perhaps the most important physical manifestation of

the Minkowski geometry in Special Relativity, namely, the behavior of free massive particles and free light pulses, i.e. ordinary matter and radiation exempt from the causal interference of other matter and radiation. The worldline of such an isolated object is a straight, i.e. a path in Minkowski spacetime whose image by a Lorentz chart satisfies a linear equation. Any two points, P and Q, of this path satisfy the inequality $|P - Q| < 0$, if the object is a free massive particle, and the equation $|P - Q| = 0$, if it is a light pulse. Presumably, free tachyons, if they existed, would follow the same law (with $|P - Q| > 0$). Thus, the spacetime does not only regulate the several avenues open (or closed) to causal connection. It also fixes the course to be followed by whatever is causally disconnected from everything else. The causalist cannot find much comfort in Special Relativity.

3

The efficacy of spacetime structure in guiding the force-free motion of matter and radiation was later cited by Einstein as a strong reason for replacing Special by General Relativity:

> The principle of inertia . . . seems to compel us to ascribe physically objective properties to the space-time continuum. Just as it was consistent from the Newtonian standpoint to make both the statements, *tempus est absolutum, spatium est absolutum*, so from the standpoint of the special theory of relativity we must say, *continuum spatii et temporis est absolutum*. In this latter statement *absolutum* means not only 'physically real', but also 'independent in its physical properties, having a physical effect, but not itself influenced by physical conditions' . . . . But . . . it is contrary to the mode of thinking in science to conceive of a thing (the space-time continuum) which acts itself, but which cannot be acted upon.[13]

In the General Theory, the spacetime geometry still provides a "guiding field" (*Führungsfeld*) for (now) freely-falling matter and radiation, but is in turn responsive to the actual distribution of matter and energy. Thus, the relation between geometry and the system of causal interactions in nature takes a new twist: the former is not merely the abstract reflection of the latter, or the norm with which the latter must comply, but is engaged in it as a full-fledged participant.

To see how this conception works, let us recall that General Relativity regards spacetime as a 4-dimensional Riemannian manifold whose metric field induces a Minkowski geometry on the tangent space at each point. The metric components g_{ij} (relative to an arbitrary spacetime chart x)[14] are the solutions of the Einstein field equations,

$$R_{ij} - \tfrac{1}{2}Rg_{ij} = -kT_{ij},$$

where the R_{ij} are the components (relative to x) of the Ricci tensor constructed from the metric (R denotes the trace $\Sigma_k R^k_k$ of the 'mixed' Ricci components), while the T_{ij} are the components of a suitable generalization of Laue's (1911) stress-energy tensor, representing the distribution of matter and (non-gravitational) energy. Let g denote the metric and consider a vector field V, defined on an open set in spacetime. $\mathbf{g}(V, V)$ is then a real-valued function (or scalar field) on the domain of V. V is said to be *timelike, null*, or *spacelike* depending on whether $\mathbf{g}(V, V)$ is everywhere negative, zero, or positive. It is usually assumed that spacetime admits a smooth global timelike vector field, whose value at each point can be used for distinguishing 'the past' from 'the future'. (This does not follow from the preceding assumptions.) A smooth curve in spacetime is *timelike* (*null, spacelike*) if it is everywhere tangent to a timelike (resp. null, spacelike) vector field. General Relativity originally postulated – and subsequently proved, under special conditions, from the Einstein field equations – that the possible worldlines of freely-falling non-spinning uncharged massive particles are the timelike geodesics of spacetime. The theory implies moreover, as shown by Laue (1920) and Whittaker (1928), that the possible worldlines of light pulses in vacuo are precisely the null geodesics. The transmission of gravitational pulses also takes place along null geodesics (Einstein, 1918). Due to the local validity of Special Relativity, massive particles must always describe timelike curves, while tachyons, if they existed, would describe spacelike curves.

Let us say that two spacetime points are *connected* if they are joined by a null or a timelike curve, and *separate* if they cannot be so joined. When thus extended to General Relativity, the relations of separateness and connectedness obviously preserve the same causal meaning they had in the Special Theory. But there can be no question of building on them a general causal theory of relativistic spacetime. Except in the special Minkowskian case, the metric of a relativistic spacetime is not determined merely by assorting its points into connected and separate pairs. A theorem proved in the early 1920's by Hermann Weyl tells us that in order to fix the metric up to a constant scale factor, something else is needed besides the classification of spacetime point-pairs according to their separateness or connectedness; namely, to single out all the geodesic paths (i.e. the ranges of geodesics).[15] The fact that the stated information contains essentially all that there is to know about the Riemannian geometry of relativistic spacetime says much about the latter's true physical import. For our two sets of data jointly

specify the geodesic paths that consist exclusively of mutually connected points, i.e. the spacetime tracks reserved for radiation and the simplest forms of ordinary freely falling matter. Indeed, the specification of these tracks is by itself sufficient – as well as necessary – for determining the metric (*modulo* a constant scale factor).[16] Thus, the relativistic spacetime structure turns out to be richer in the general case than the abstract scheme of viable causal links articulated in the relations of separateness and connectedness; but it is just as strong as is required for it to play its appointed role as a guiding field of matter and radiation. Therefore – we may conclude – it is this role that is the *raison d'être* of spacetime geometry.

The idea that a geometric structure might contribute to the actual shaping of physical events is so alien to our received modes of thought that, from the advent of General Relativity, there has been a tendency to explain this idea away, to reduce spacetime geometry to the role of a mere mediator between interacting material things, perhaps no more than an aid to calculation. Even Einstein himself may have yielded to this tendency in the earlier days of the theory, when he still bowed to Mach's authority. This interpretation is apparently inbuilt in the Einstein field equations, for the matter distribution represented by the T_{ij} on the right-hand side is obviously meant to be the source of the field represented by the g_{ij}, through which gravitational action is exerted on test particles. However, as de Sitter showed already in 1917, there are sourceless solutions (with the T_{ij} identically zero) of the field equations – apart from the Minkowski solution, which may be dismissed as trivial. It can indeed be argued that such solutions are merely a consequence of the formal mathematical properties of the Einstein equations, but have nothing to do with the real meaning of their terms in actual physical applications. But even if we grant this allegation,[17] the fact remains that an equation is a two-sided affair, and it is therefore more sensible to say that the Einstein equations express a mutual dependence of matter and geometry, rather than a total subordination of the latter to the former. Indeed, there are physically significant solutions of the Einstein equations in which the metric field can hardly be said to mediate between its sources and a test particle subjected to its guidance, for the prevailing conditions do not allow any gravitational action of the sources on that particle, or permit only such action as would not be sufficient to account for the particle's predicted behavior.

Consider, for example, the Schwarzschild (1916) vacuum solution, applied in all classical tests of the theory. The solution does not follow from a detailed knowledge of the matter distribution, but is worked out from global

assumptions concerning the metric field – namely, that it can be decomposed into isometric, asymptotically flat, spherically symmetric spacelike slices.[18] One naturally assumes that the field's source surrounds the center of symmetry. But there is no indication as to how the source might have produced the field, which, being static, bears of course no trace of its origin. Indeed, if the source is entirely contained within the so-called Schwarzschild radius, no gravitational action can ever irradiate from it into the field beyond this radius, so that the field and any test particle traversing it ought to be regarded as causally independent from the source. Matter falling into the 'black hole' determined by the Schwarzschild radius becomes dynamically idle with regard to the space outside. (This is apt to be overlooked if one still thinks in terms of Newtonian instantaneous action-at-a-distance. It shows, by the way, that the fact that a Schwarzschild field will persist unchanged if its source collapses isotropically under its own weight does not imply that the source remains in hiding inside the singularity at the center of symmetry, instead of vanishing there right out of existence. In classical General Relativity, anything that sinks into such a singularity is withdrawn forever from the economy of nature.)

It may be objected to the preceding discussion that the Schwarzschild field is no less unreal than the sourceless solutions that we chose to ignore. It is valuable only as a first approximation to the field surrounding a big star, which is, of course, virtually static and spherically symmetric, and goes over to near flatness in the vast interstellar spaces. But all embodiments of the Schwarzschild solution in the real world must have been generated at some time or other by dynamical processes issuing from their respective material sources. To avoid such objections we must turn to the cosmological solutions of the Einstein equations, which embrace all events within their scope. Let us take a look at the earliest and simplest: the family of solutions discovered by Alexander Friedmann (1922; 1924), of which the 1917 Einstein (static) and de Sitter (sourceless) solutions are limiting cases. As is well known, the Friedmann solutions provide a framework within which one can readily account for the systematic shift towards the red of the spectra of distant galaxies first observed by Slipher, and for the isotropic microwave black-body radiation discovered by Penzias and Wilson. To achieve his results, Friedmann postulated that matter can be represented as a pressureless fluid and made the following assumptions concerning, as he put it, "the general, so to speak geometrical character of the world":[19]

(i) Spacetime admits a global time coordinate whose parametric lines

are the integral curves of the worldvelocity field of matter (the time coordinate measures proper time along each worldline of matter).

(ii) The hypersurfaces orthogonal to the said integral curves are maximally symmetric 3-dimensional subspaces of spacetime, and hence Riemannian spaces of constant curvature (relative to the positive definite metric induced in each of them by the spacetime metric).

Einstein's field equations rigorously imply that the particles of a pressureless fluid describe timelike geodesics.[20] These geodesics are, of course, the integral curves of the worldvelocity field of matter mentioned under (i), and therefore they constitute a congruence that fills the entire Friedmann universe. Except in some special cases (which include the static Einstein and the de Sitter solution), the geodesics of the said congruence are incomplete,[21] and are focussed in the directions of decreasing and increasing time (expanding and recontracting Friedmann universe), or, at least, in one of these directions (the universe then expands from a point to infinity, or contracts from infinity to a point). Since timelike geodesics can be parametrized by proper time, matter in a typical Friedmann universe has only a finite past and/or a finite future. The feature of the Friedmann universes that is most relevant to our present discussion is the existence of horizons.[22] Let p denote a particle and $S(p)$ the set of spacetime points from which a light signal – or a gravitational pulse – can be sent to p. The *event horizon* of p is the boundary of $S(p)$. Let E denote an event in the history of p, and let $S(E)$ be the set of matter worldlines from which a light signal – or a gravitational pulse – can be sent to p so that it arrives before E. The boundary of $S(E)$ – in the quotient space of spacetime by the congruence of matter worldlines – is the *particle horizon* of E. If $S(p)$ is not empty there are spacetime points from which no electromagnetic or gravitational influence can ever be received by p; if $S(E)$ is not empty there are particles of matter from which no such influence can arrive in p before E. Friedmann universes, and other more realistic world models related to them, have non-empty particle horizons.[23] Consider a particle p in a Friedmann universe that expands from a point. Let $t(E)$ be the time of an event E at p, as given by the standard Friedmann global time coordinate. As $t(E)$ decreases, approaching the greatest lower bound of time, fewer and fewer particles are found within E's particle horizon, until, in the limit, there remains p alone. In the earliest times of cosmic evolution each particle has only had opportunity to suffer gravitational influence from or exert it on its nearest neighbors. Therefore, it makes little sense to explain the global structure of the Friedmann field, which shapes the worldlines of matter, by the gravitational

interaction of each material particle with all the rest. Mutual communication between the several parts of matter is gradually attained, within the framework laid down by the metric field.

To ascribe such a dramatic dynamic effect as the expansion of the universe to the inherent evolutionary tendencies of the spacetime geometry will probably be distasteful to many philosophers. However, in the context of relativistic cosmology, I do not see how one could avoid it. When John Earman wrote some time ago that in General Relativity "the deviation of the world lines of a system of test particles is caused by the curvature of space-time",[24] Bas van Fraassen remarked that "the assertion that space-time causes deviations in world lines is a category mistake."[25] Van Fraassen would doubtless be right if the spacetime curvature were expected to deviate world-lines in the way in which a policeman might deviate traffic. But all that Earman meant to say was that, given the mathematical fact of Jacobi's equation, a non-zero spacetime curvature at a point entails a change in the rate at which a timelike geodesic through that point approaches or recedes from nearby timelike geodesics. Now, physicists ordinarily say, in an altogether similar sense, that the electromagnetic field deflects a beam of charged particles, and nobody has ever accused them of making a category mistake when they say so. Indeed, as soon as Riemann succeeded in characterizing a broad family of geometric structures – including the familiar Euclidean and the then still unknown Minkowski geometry – by means of a tensor field, i.e. by the same type of mathematical object usually employed for representing a field of force, the way was open for incorporating physical geometry into the dynamical transactions of nature. However, even if the tensor field representation of metric relations in spacetime does furnish us the means of understanding and accurately formulating their interdependence with matter and the forces of nature, we ought not to jump to the conclusion that General Relativity has overcome the duality of cause and structure, power and form, that we encountered in Newtonian dynamics. The very concept of a tensor field presupposes a manifold structure on which the field is defined. The dynamical interplay of fields governed by the Einstein equations requires a smooth, topologically shaped arena in which to evolve.

Several attempts have been made to construct the topology of relativistic spacetime from that of causal chains, which is assumed to agree at least locally with the topology of the linear continuum. (The latter can, of course, be defined in terms of the relation of causal precedence, if we introduce the necessary denseness and continuity postulates which many believe to be empirically vindicated by our direct awareness of our own lives.) Very

interesting in this connection is the *path topology* of spacetime, defined by Hawking, King and McCarthy (1976).

In order to define it I must introduce a few terms. Let M be an arbitrary relativistic spacetime. A smooth curve in M is *future-directed* if the tangent vector at each point of the curve lies within the future lobe of the local null cone. Consider a point $P \in M$ and a subset $U \subset M$. The *future* $I^+(P, U)$ of P *relative* to U is the set of all points in U that can be reached from P by a future-directed smooth curve of finite extent whose range is wholly contained in U. We abbreviate $I^+(P, M)$ to $I^+(P)$ and call it simply the future of P. Past-directed smooth curves, the past $I^-(P)$ of a point $P \in M$ and its past $I^-(P, U)$ relative to a subset $U \subset M$ are similarly defined. A neighborhood U of a point $P \in M$ is said to be *normal* if the exponential mapping Exp_p maps a neighborhood of the zero vector in the tangent space at P diffeomorphically onto U. U is *convex* if any two points $Q_1, Q_2 \in U$ are joined by a geodesic, unique up to reparametrization, whose range lies wholly within U. A continuous curve $\gamma: J \longrightarrow M$ is future-directed at $t_0 \in J$ if there is a connected neighborhood H of t_0 in J and an open normal convex neighborhood U of γt_0 in M such that, for any $t \in H$, $\gamma t \in I^-(\gamma t_0, U)$ if $t < t_0$ and $t \in I^+(\gamma t_0, U)$ if $t_0 < t$. The continuous curve γ is future-directed if it is future-directed at every $t \in J$. Past-directed continuous curves are defined analogously. A continuous curve that is either past or future directed is said to be *timelike*. It is not hard to see that smooth timelike curves as defined earlier are also timelike in this extended sense. Consider now the collection S of all timelike paths (i.e. all ranges of timelike continuous curves) in M.[26] Each element of S is a one-dimensional subspace of M. This implies that if U is a 'coordinate patch' (the domain of a chart) of M and $P \in S$, $U \cap P$ is open in P. The *path topology* of M is the strongest topology that induces in each timelike path the same subspace topology as the standard manifold topology of M. In contrast with the latter (which is generated by the coordinate patches) the path topology is uniquely determined – through the requirement of maximal strength – by the one-dimensional topologies of the paths themselves, i.e. by neighborhood relations on each possible worldline of matter. David Malament (1977) proved that a permutation f of M that preserves the path topology (i.e. such that both f and its inverse f^{-1} map open sets onto open sets) will also preserve the standard manifold topology of M, its differentiable structure and the distinction between connected and separate point-pairs.[27]

Two more theorems, also due to Malament (1977), further illustrate the links that can be established between topology and causality in a relativistic

spacetime. If f is any permutation of the spacetime M such that both it and its inverse f^{-1} map timelike paths onto timelike paths, f preserves the standard manifold topology of M. Moreover, if M is both past- and future-distinguishing, i.e. if no two distinct points of M share the same future or the same past, then any permutation f of M which maps the future and the past of each $P \in M$ respectively onto the future and the past of its image $f(P)$ will also preserve the standard manifold topology of M. The latter is therefore determined, in a vast family of cases, by the scheme of possible worldlines of matter.[28]

Malament's results testify to close bonds between the fundamental neighborhood relations of spacetime points and the possible causal connections between events at them. But one can hardly claim that the former have thereby been reduced to the latter. It is just as well that they haven't. For when the turn comes to explain what is meant by causality, philosophers of the most diverse persuasion mention spatial and temporal contiguity as its surest and least problematic ingredient. Thus, David Hume's first "rule by which to judge causes and effects" is that "the cause and the effect must be contiguous in space and time" (*Treatise*, I. iii. 15, p. 173). And C. J. Ducasse's celebrated anti-Humean analysis of the causal relation is wholly stated in spatial and temporal terms.[29] Obviously both Hume and Ducasse had a feeling that our ideas of time and space were clearer and less in need of explication than our idea of cause. If, as I sense, they had the right feeling, the causal theory of spacetime would be a notorious example of *elucidatio per obscurius*.[30]

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NOTES

[1] Descartes, *AT*, VI, 62 (*Discours de la méthode*).

[2] "Omnis enim philosophiae difficultas in eo versari videtur, ut a phaenomenis motuum investigemus vires naturae, deinde ab his viribus demonstremus phaenomena reliqua" (Newton, *Principia*, p. 16).

[3] Newton (1962), p. 103.

[4] *Ibid.*, p. 99.

[5] Reichenbach (1928), p. 307; (1958), p. 269.

[6] Reichenbach (1928), pp. 307 f.; (1958), p. 268.

[7] This implies, in particular, that I am paying no heed to the special context in which Reichenbach made these statements, in connection with his astounding claim that "in the most general gravitational fields" countenanced by General Relativity "all of the

metrical properties of the space-time continuum are destroyed" (because all rigid rods and clocks are shattered where the spacetime curvature is large enough!).

[8] Einstein (1905), pp. 893f.

[9] Let A and B be the matrices of two Lorentz transformations. Then $(AB)^{\mathrm{T}} H(AB) = B^{\mathrm{T}}(A^{\mathrm{T}}HA)B = B^{\mathrm{T}}HB = H$. Recalling that the transpose of the inverse matrix A^{-1} is the inverse of the transpose A^{T}, we verify that $(A^{-1})^{\mathrm{T}}HA^{-1} = (A^{\mathrm{T}})^{-1}A^{\mathrm{T}}HAA^{-1} = H$, so that the inverse of A is also the matrix of a Lorentz transformation. It is obvious, moreover, that the identity matrix I meets the condition $I^{\mathrm{T}}HI = H$.

[10] It can be readily seen that $|P - Q|$ does not depend on the choice of the particular Lorentz chart in which the coordinates of P and Q are (P^i) and (Q^i). For let $(\bar{P}^i)$ and $(\bar{Q}^i)$ be their respective coordinates in another Lorentz chart related to the former by the transformation $x^i \mapsto \Sigma_j A_{ij}x^j + k^i$, where the k^i are constants and (A_{ij}) is the matrix of a Lorentz transformation. We have then that

$$\begin{aligned}
&\Sigma_{ij}\eta_{ij}(\bar{P}^i - \bar{Q}^i)(\bar{P}^j - \bar{Q}^j)\\
&= \Sigma_{ijkh}\eta_{ij}A_{ik}A_{jh}(P^k - Q^k)(P^h - Q^h)\\
&= \Sigma_{ijkh}A^{T}_{ki}\eta_{ij}A_{jh}(P^k - Q^k)(P^h - Q^h)\\
&= \Sigma_{kh}\eta_{kh}(P^k - Q^k)(P^h - Q^h) = |P - Q|.
\end{aligned}$$

[11] Since, according to some authors, causal relations themselves are modal too, one might be tempted to speak here of 'second modalization'.

[12] Minkowski (1909), p. 104: "Um nirgends eine gähnende Leere zu lassen, wollen wir uns vorstellen, dass aller Orten und zu jeder Zeit etwas Wahrnehmbares vorhanden ist."

[13] Einstein (1956), pp. 55f.

[14] Let x^i be the ith coordinate function of x; denote by $\partial/\partial x^i$ the vector field whose integral curves are the parametric lines of x^i. Then, if g stands for the metric tensor, $g_{ij} = \mathrm{g}(\partial/\partial x^i, \partial/\partial x^j)$. (A (0, 2)-tensor field such as g assigns to each pair of vector fields defined on an open subset U of spacetime a smooth real-valued function on U.) The tensor components R_{ij} and T_{ij} that occur in the Einstein field equations are defined analogously.

[15] Weyl (1921).

[16] Ehlers, Pirani and Schild (1972).

[17] Such philosophically motivated restrictions on the possible physical meaning of mathematical objects are often unquestionable (as when an electrical engineer discards the imaginary part of the complex numbers that turn up in the end result of his calculations); but they can also severely impair the heuristic power of a theory. (Just think of the consequences of dismissing the negative energy solutions of Dirac's equation as 'physically meaningless'.)

[18] We now know that the Schwarzschild metric is essentially the one and only spherically symmetric solution to the Einstein vacuum field equations, so that Schwarzschild's further assumptions are redundant. See Birkhoff (1923), p. 255; Petrov (1969), p. 360; Hawking and Ellis (1973), App. B; cf. Adolf Grünbaum's apt reminder (1973, p. 840)

[19] Friedmann (1922), p. 378.

[20] Einstein and Grossmann (1913), p. 10. For a proof, see Rindler (1977), p. 182.

[21] Like any (parametrized) curve, a geodesic is a continuous mapping of an interval of the real number field into spacetime. A timelike geodesic γ is said to be incomplete

if its domain is a proper subset of the real number field and γ is not the restriction to that domain of a timelike geodesic defined on the entire real number field.

[22] Rindler (1956).

[23] MacCallum (1971).

[24] Earman (1972), p. 84.

[25] van Fraassen (1972), p. 93.

[26] Hawking *et al.* (1976) actually defined the path topology for so-called strongly causal spacetimes (i.e. spacetimes in which every point has a neighborhood that no timelike or null curve penetrates more than once). I should note also that they use 'curve' for what I call 'path', and vice versa.

[27] Strictly speaking, what Malament showed was that, if $f\colon M \longrightarrow M$ is a homeomorphism of the path topology, then f is smooth and the spacetime metric g is conformal with its pull-back $f^*\mathrm{g}$ (in other words: $f^*\mathrm{g} = \omega^2\mathrm{g}$, where ω is a smooth real-valued function on M; this condition evidently implies that both g and $f^*\mathrm{g}$ determine the same null-cone in the tangent space at each point of M).

[28] More precisely: $P \ll Q$ if there is a timelike future directed curve γ: $[a, b] \longrightarrow M$, such that $\gamma(a) = P$ and $\gamma(b) = Q$.

[29] Ducasse (1926) defined causality as follows:

Considering two changes, C and K (which may be either of the same or of different objects), the change C is said to have been sufficient to, i.e. to have caused, the change K, if: 1. The change C occurred during a time and through a space terminating at the instant I at the surface S. 2. The change K occurred during a time and through a space beginning at the instant I at the surface S. 3. No change other than C occurred during the time and through the space of C, and no change other than K during the time and through the space of K.

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